Chapter 22

Quantum Field Theory

22.1 Introduction

So far we have studied the properties of photons and electrons primarily as non-interacting single particles. The many particle states that we discuss are just collections of non-interacting single particle quantum systems. It was Einstein's great discovery to realize that these constituents of light are the substance of things we call light but again his detectors deal with only the single photon interactions. Granted, given Bell's Theorem and the Young's Double Slit experiment, these are entities that are very different from those that are encountered in a Newtonian world. At the same time, we realize that we have to develop a picture based on photons that adequately describes the many wavelike properties that are associated with light, the field properties. The theory that does that is quantum field theory. We want to make a quantum theory of the electromagnetic field which preserves its classical success and yet meets the requirements imposed by Planck and Einstein.

The electromagnetic field is a rather complex field; it is a combination of two vector fields with a rather complex dynamics. For this reason, we will first discuss a simpler field, the stretched string. It possesses the essential features of a field theory and, at the same time, we can understand the dynamical properties in sufficient detail to be able to construct a quantum theory. The problem though is that nature has chosen not to have any known system manifest this exact pattern of behavior. We will construct it by realizing that the phenomena that we identify with the field nature of light is characterized by energies that are large compared to $\hbar\omega$ and therefore states with many photons. This is consistent with our study of the quantum oscillator, see Sections 19.9.2 and 19.11.2, which indicated that to recover classical motion, we needed states composed of several stationary states. These are the principle goals of this Chapter.

Our first problem will be to describe the many photon state. This is actually a subtle construction and will lead us in to a new definition of the identity of these particles. Actually, we are laboring to develop a formalism that simply leads to the strange counting that Planck originally discovered. Once we have a coherent description of light, we will add the fundamental charged particles, electrons, and review the theory called quantum electrodynamics. This is a complete theory of the world of photons and electrons and describe successfully all the phenomena that emerges in systems with only these constituents. It is the most successful theory ever developed. It agree with experiment to one part in 10^15, an incredible precision. This is the theory that is called Quantum ElectroDynamics or QED. A detail look at this theory requires that we understand processes at a fundamental level. The most complete language for describing this theory is based on an analysis using Feynman diagrams. With the experience of using these diagrams, we can develop the current language for the description of all the fundamental processes that have as yet been observered. We will cover also one of the great theorems of modern physics, the spin statistics theorem.

22.2 The Many Photon State

Many things locally transfer different amounts of energy and momentum and other things. They do this locally in both space and time. The example that we have been dealing with is light and, from what we know from Einstein, the transfer, when we use monochromatic light, of some physical property is done discretely. For example, the energy is an integer multiple of $\hbar\omega$ where ω is the radian frequency of the light. Similarly for the momentum which comes in units of $\frac{h}{\lambda}$ where λ is the classical wavelength. For the angular momentum the unit is $\hbar$.

The energy is related to the time evolution of the state and thus there is a frequency identified, $\omega = \frac{\epsilon}{\hbar}$. This frequency is related to the classical frequency. I remind you though that in the definite energy state of a quantum system nothing is moving back and forth.

From the classical relationships we know that there is a relationship between the energy and momentum, $\epsilon =\mid \vec{p} \mid c$.

The polarization of the light was known from the classical case to be related to the angular momentum of the light. The photon is said to have an intrinsic angular momentum L $= \hbar$. In fact we can do experiments that measure the angular momentum transferred by the absorption of photons. I remind you that classically the polarization comes from the vector nature of the field.

Let us consider the case of light of only one frequency, ω, and therefore the photons have energy $\hbar\omega$, and thus also a momentum $p = \frac{\hbar}{\lambda}$, and some intrinsic angular momentum state. We want the states to be amplitudes and the multiphoton state comes from putting several photons in the state.

22.3 The Quantum Stretched String

In the case of the stretched string, we saw that the string can be describes as an infinity of independent oscillators, one for each of the normal modes. Each of these modes has a frequency of the normal mode, $\sqrt{\frac{T}{\rho}}\frac{\alpha\pi}{L}$, where T is the tension in the string, ρ is the mass per unit length, L is the length of the string, and α is an integer from 1 to ∞ and also labels the mode.

We saw that a quantum oscillator has definite energy states and that these have a definite frequency and the energy is an $\left(n + \frac{1}{2}\right)\hbar\omega$, where ω is the frequency of the oscillator.

The general state is thus a system in which for each mode there is a number of excitations, $\{n_i\}$.

$$| \ \rangle = | n_1, n_2, n_3, \cdots, n_m, \cdots\rangle \tag{22.1}$$

Each state like this will have a definite energy

$$\begin{aligned} E &= \left(n_1 + \frac{1}{2}\right)\hbar\omega_1 + \left(n_2 + \frac{1}{2}\right)\hbar\omega_2 + \left(n_3 + \frac{1}{2}\right)\hbar\omega_3 + \cdots \\ &\quad + \left(n_m + \frac{1}{2}\right)\hbar\omega_m + \cdots \\ &= \sum_{m=1}^{\infty}\left(n_m + \frac{1}{2}\right)\hbar\omega_m \end{aligned} \tag{22.2}$$

The state that has an ambiguity. All the states that have the excitations in different orders are the same. All excitations of the same mode are identical. This is a new definition of identical. Using this definition of identical you recover the magic counting that Planck needed to get the black body distribution

These states are orthogonal

22.4 The field

In the oscillator, we saw that the displacement of the mass required a superposition of many excitations. The state with a definite amplitude is not a state with a definite number of excitations. This is similar to the problem that we had with the states of polarization in light. These are incompatible measurements.

You can show that the expected value of the field in any definite energy state is zero. This is the same situation that we had in the oscillator and thus makes sense.

Also we have the same situation as in the oscillator. The definite energy state has a field^2 that is not zero.

22.5 Elementary Particles

These are the things that transfer discrete amounts of energy and momentum and other things. The example that we have been dealing with is the photon. It has a definite energy and momentum. It also has some intrinsic directional information as is shown by the polarization. The energy is related to the time evolution of the state and thus there is a frequency identified,$\omega = \frac{\epsilon}{\hbar}$. I remind you though that in the definite energy state nothing is moving back and forth.

In an empty space the thing that we have called the mode label is the momentum.

From the classical relationships we know that there is a relationship between the energy and momentum, $\epsilon =\mid \vec{p} \mid c$.

The polarization was known from the classical case to be related to the angular momentum of the light. The n is said to have an intrinsic angular momentum L $= \hbar$. I remind you that the polarization comes from the vector nature of the field. If we had a given polarization and then separated the different values of the polarization at a new angle θ relative to the original direction, the probabilities shifted to $n \cos^2 \theta$ etc..

The field takes on non-zero values when you have a carefully arranged state of many excitations. Systems with a classical field can have states with a large number of excitations in the same mode. These are things like ns and phonons.

An example of another particle is the electron. There is an associated field and the excitations are the particle. One difference is that the electron has mass. The mode label is the again the momentum. For a slowly moving electron, we have $\epsilon = \frac{p^2}{2m}$.

The electron also has a polarization and there is even a device like the calcite that separates the states. The difference is that if you reorient the apparatus the probabilities go as $n \cos^2 \left(\frac{\theta}{2}\right)$. Thus we say that the electron has an intrinsic angular momentum and the value is $\mathrm{L} = \frac{\hbar}{2}$.

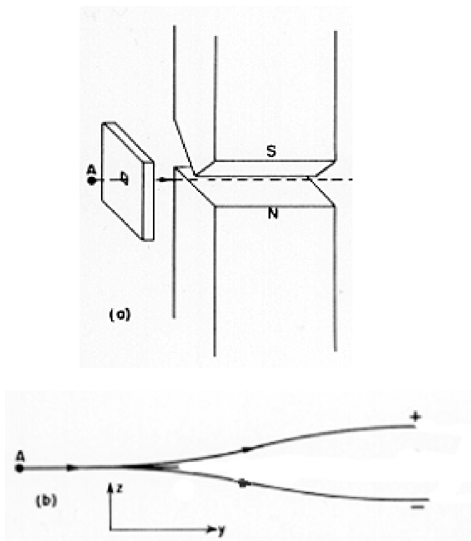

Figure 22.1: A diagrammatic representation of the Stern Gerlach apparatus. In the upper figure a beam of electrons passes from point A through an aperture and between the poles pieces of a magnet with an inhomogeneous field. In the lower part of the figure is how the beam is split into two beams one with spin up, labeled +, and the other with spin down, labeled −.

22.6 Fundamental Processes

Not only does the particle follow all possible paths, it undergoes all basic processes. This is best illustrated diagrammatically. Before going into the real processes relevant to electron and photons, it is worth examining some simple illustrative processes. Consider a drunk going home after an evening of partying. If he was a quantum drunk, his probability of getting home would be given by an amplitude from the party to home, $\Psi(\text{home}, \text{party})$, and his probability of being home after the party is $\psi^*(\text{home}, \text{party})\Psi(\text{home}, \text{party})$. But the evolution of his trip home is to stop at all distracting places on the way. Thus the amplitude for getting home from the party is

$$\begin{aligned}\Psi(\text{home, party}) &= \Psi_0(\text{home, party}) + \Psi_0(\text{home, Alice's})g_0(\text{Alice's})\Psi_0(\text{Alice's, party}) \\ &+\Psi_0(\text{home, Bob's})g_0(\text{Bob's})\Psi_0(\text{Bob's, party})\ldots \\ &+\Psi_0(\text{home, Alice's})g_0(\text{Alice's})\Psi_0(\text{Alice's, Bob's})g_0(\text{Bob's})\Psi_0(\text{Bob's, party})\ldots \\ &+\Psi_0(\text{home, Alice's})g_0(\text{Alice's})\Psi_0(\text{Alice's, Alice's})g_0(\text{Alice's})\Psi_0(\text{Alice's, party})\ldots \\ &+\ldots,\end{aligned}$$

where $\Psi_0(\text{anywhere, anywhere})$ is the fundamental amplitude for going from anywhere to anywhere and $g_0(\text{newplace})$ is the strength of getting into the place once there.

For instance, in the action for a charged particle there has to be a term that has both the particle terms, now a field for the electron and the electromagnetic field. This is because a charged particle is the source of an electromagnetic field and the electromagnetic field also produces changes in the motion of the electron. It is just a fact of life that all matter is made up of these fundamental constituents and these quantum properties are the basic operating procedures.

In the action formulation of physics, you have to introduce all effects through an action term. Therefore there is a generalization of the action to allow for an interaction. All interactions come from a term in the action. The neat thing about this is that it is a generalization of the old action reaction law.

$$\begin{aligned}\text{Action}_{\text{Total}} &= \text{Action(variables particle 1)} + \text{Action(variables particle 2)} \\ &+\text{Action(variables particle 1, variables particle 2)}\end{aligned} \qquad (22.3)$$

For example, the electron and the electromagnetic field have to have a term in the action that looks like

$$S = -mc^2\int_{t_1}^{t_2} d\tau - q\int_{t_1}^{t_2}[\phi(x,y,z,t) - \vec{v}\cdot\vec{A}(x,y,z,t)]dt \qquad (22.4)$$

See Feynman lecture on action.

In the graphical language that we are developing there is a fundamental process in which an electron becomes an electron and a photon, see Figure 22.2. If you have that process you also have a photon and an electron turning into an electron and an electron and a positron, an anti-electron, turning into a photon, see Figure 22.3, and a photon turning into an electron positron pair see Figure 22.4. I will return to this issue and the positrons when we have more of the material developed.

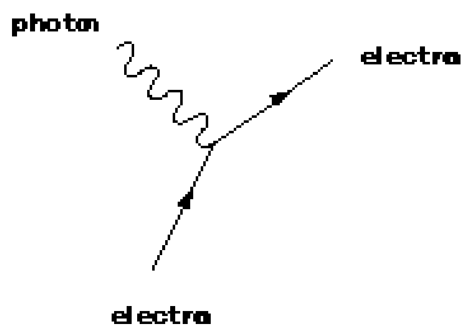

Figure 22.2: A space-time or Feynman diagram of the fundamental electromagnetic interaction. An electron enters at some time at the bottom of the figure. At a later time, it changes its velocity and emits a photon.

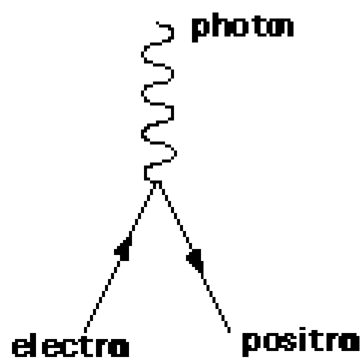

Figure 22.3: A space-time diagram depicting the annihilation of an electron positron pair into a photon.

None of these processes can occur and conserve energy momentum. They are virtual. We had the first virtual reality!

The basic point is that all fundamental processes occur locally, stochastically and instantaneously. In addition to following all paths, all processes occur also. Each of these enter through the action. All interactions have an effect on the action.

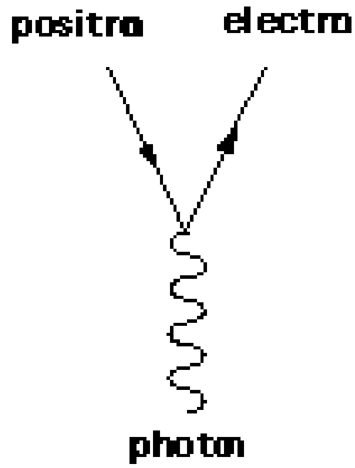

Figure 22.4: A space-time diagram depicting the the process by which a photon is converted into an electron positron pair.

22.7 Feynman Diagrams

22.7.1 Introduction

These are the techniques that have been developed to compute the full evolution of quantum systems. The basic rules are the same as in our original discussion of the Einstein photo electric effect, 19.3 on page 423